Liability Situations with Joint Tortfeasors

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Given a tort that involves several tortfeasors, an allocation scheme attributes to each of them that part of the damage that reflects their responsibility. We consider successive torts, i.e. torts that involve a causality chain, and show that simple axioms, that are well known in the law of tort, uniquely define an allocation scheme. We show that this scheme incentivizes agents to satisfy a standard of care, creating an efficient prevention of accidents. We further describe the unique rule according to which a liability situation has to be adjusted after a partial settlement such that incentives to settle early are created.

Keywords: tort law, successive tortfeasors, allocation scheme, negligence, deterrence

JEL: K13, C71, D63

1 Introduction

A driver negligently hits a pedestrian and a physician negligently treats, thereby aggravating, the pedestrian's injury ... if the first tortfeasor had not acted tortiously, the entire injury to the plaintiff – initial plus incremental damages – might have been avoided, whereas if only the second injurer had been nonnegligent only the incremental damages could have been avoided (Landes and Posner, 1980). The natural question arises: how much should each tortfeasor pay the victim in order to compensate for the damage? The principles of attributing damages to several tortfeasors depend on the legal regimes; we briefly summarize the discussions in Kornhauser and Revesz (2000) and Wall (1986) that are relevant for our purposes. Under *non-joint liability*, a damage must be attributed to each defendant and each defendant has to pay a compensation for the damage attributed to him. Hence there is a direct need for an allocation scheme that allocates damages to tortfeasors. Under *joint liability*, there is no direct need for such a scheme as the plaintiff can recover the full damage from any defendant he prevails (subject to not being overcompensated). Nevertheless, a plaintiff can settle with some of the defendants. Under the *pro tanto* set-off rule the damage attributable to the remaining defendants is then reduced by the the settlement amount, whereas under the *apportioned set-off* rule it is reduced by the damage attributable to the settling defendants. Hence while in the first case an allocation scheme can serve as an anchor for settlements, in the second case it is necessary to calculate the set-off rule.

But: how should such an allocation scheme be designed? There are normative principles, axioms, such a scheme should satisfy (Dehez and Ferey, 2013): it should be consistent with a rule of liability, i.e. agents should have to pay only if they are liable according to such a rule (Shavell, 1980). It should ensure that all damages that have been caused negligently, should be recovered. In case of successive torts, the amount attributed to an agent should not depend on the damages that preceded his involvement. We formalize some of the rules that have been proposed in the literature on the law of tort, and we show that there is only one allocation scheme that satisfies all of them.

Another approach focuses on the efficient prevention of accidents. Setting a standard of care means balancing out the (expected) costs of an accident and the cost of reducing the probability that it occurs. After the right standard of care has been found, agents have to be incentivized to act accordingly (cf. Landes and Posner, 1980). We define a game where each agent can choose how much to invest into the prevention of accidents and show that every allocation scheme that satisfies two very easy axioms incentivizes agents to invest an efficient amount.

In a liability situation with multiple tortfeasors some of them may settle with the plaintiff. Both under apportioned set-off and pro tanto the payments of the remaining agents have to be adjusted – ideally in a way that incentivizes all agents to settle early. For the allocation scheme we provide there is a unique key according to which the settlement amount as to be distributed between the incremental damages in the causality chain.

Throughout the paper we will use the following numerical example to illustrate our points: two drivers have an accident and a pedestrian who stands nearby is injured, a damage of $100,000. A physician negligently treats the pedestrian causing an incremental damage of \$900,000. The three tortfeasors have to compensate for an overall damage of \$1,000,000.

The article is structured as follows: in Section 2 we focus on the normative approach and develop the unique allocation scheme with all desired properties. In Section 3 we provide a minimal requirement on the allocation scheme as to achieve the efficient prevention of accidents. The proper adjustment of damages after settlements is discussed in Section 4. Section 5 closes the article with a brief discussion. The mathematics is postponed to the appendix.

2 Obtaining the Compensation Payments

Throughout the paper we consider liability situations where groups of agents (tortfeasors) subsequently cause (incremental) damages to a plaintiff, and where the damage caused by any but the first group is only possible because of the behavior of the previous groups. We say that a tortfeasor *directly* causes an incremental damage if he belongs to the group that caused that damage, and we say he *indirectly* causes a damage if the incremental damage was caused in the sequel of his group's action. The plaintiff is not a member of any of these groups, and each other agent belongs to exactly one group, i.e. we do not consider situations where an agent contributes through multiple own acts.

An *allocation scheme* is a rule that specifies for *any* liability situation for how much of the overall damage each tortfeasors must compensate the plaintiff. The literature on tortfeasors suggests several principles such an allocation scheme should follow. We shall precisely formulate them here as axioms and investigate what allocation scheme satisfies them all simultaneously.

In order to be compensated for any damage by a defendant, the plaintiff has to prove that the damage lies in the scope of liability of the defendant, and that there is an applicable rule of liability under which the defendant has to compensate the plaintiff (Shavell, 1980). We shall not discuss the scope of liability in this article as such a discussion would distract from the point we want to make. The only thing that shall be decided is whether or not there is an applicable rule of liability. For the arguments of this section it is not necessary to specify this rule; for simplicity assume that the negligence rule is in place. We will have a closer look into the "optimal" rule of liability in Section 3. The first two axioms an allocation scheme should satisfy are easily formulated and do not need much discussion.

Axiom 1. A defendant's compensation payment is strictly positive if and only if he negligently caused a strictly positive damage.

Axiom 2. The sum of the compensation payments covers exactly all damages that are caused negligently, either directly or indirectly.

To keep things simple we shall assume that there is no discrimination between different degrees of negligence; either a tortfeasor was negligent or not. In particular, there should be no discrimination between negligent tortfeasors who caused the same incremental damages.

Axiom 3. The compensation payments of two defendants who (i) are both negligent and (ii) entered the scene together are equal.

In liability situations with only one tort, the axioms we have formulated so far are sufficient to pin down a unique allocation scheme.

Theorem 2.1. *There is a unique allocation scheme that satisfies Axioms 1-3 on all liability situations that are caused by simultaneous torts. It distributes the full compensation payment equally among the negligent agents.*

This result is hardly surprising, and the far more challenging task is the generalization of this allocation scheme to situations in which not all tortfeasors acted simultaneously. A first step is the following "uncontroversial principle" (Landes and Posner, 1980) in the law of tort.

Axiom 4. A defendants compensation payment does not depend on the damages that precede the damage he has caused.

In some situations a tortfeasor might actually not have caused a *direct* damage, but only opened the door for future damages: suppose that in the initial example the pedestrian was not injured in the accident, but during the check-up by a paramedic who appeared at the scene. In this case the car drivers would still be (partly) responsible for this injury. So, they would be treated as if they had caused the injury together with the paramedic. The following axiom captures this reasoning.

Axiom 5. Any agent who did not cause a damage but made the subsequent damages possible is treated as if he belonged to the first subsequent group that actually caused a damage.

These five axioms are, in fact, sufficient to uniquely specify an allocation scheme.

Theorem 2.2. *There exists an allocation scheme that satisfies Axioms 1-5, and this allocation scheme is unique. It proceeds as follows. All damages that are directly or indirectly caused by at least one negligent agent will be covered. Each incremental damage is equally distributed between all negligent agents that are directly or indirectly responsible for it.*

Interestingly the axioms in the foregoing theorem do not only uniquely define an allocation scheme; each of them is needed to guarantee uniqueness: whenever one of the axioms is left out, there are several allocation schemes that satisfy all the others.

When we apply this allocation scheme to our initial example, we see that the damage of $100,000 has to be equally divided between the two drivers, and the incremental damage of $900,000 has to be divided between all three tortfeasors. Hence, the compensation payments will be $350,000 for each driver and $300,000 for the physician.

The allocation scheme we present here is well known in economics, in particular in the field of cooperative game theory. Here it is known as the *Shapley value* (Shapley, 1953b). Bearing this in mind there are several generalizations one can think of. First, the plaintiff might be (partly) responsible for (some of) the damages. In this case one can obtain a similar result if one includes him as a tortfeasor in the liability situation. His net compensation payment is then the amount of all (negligently caused) damages reduced by the payment the allocation scheme allocates to him. Second, there might be a discrimination between tortfeasors depending on their degree of negligence. In this case Axiom 3 must necessarily be dropped. However, if one assume the degree of negligence is independent of the monetary value of a damage, one obtains (together with the other axioms) a family of allocation rules that only depend on the degrees of negligence of all tortfeasors. Such an allocation scheme would correspond to the weighted Shapley value (Shapley, 1953a; Kalai and Samet, 1987).

3 Efficiency and Deterrence

Suppose that taking care, i.e. avoiding accidents, is costly to agents: taking more care is more expensive and results in a lower probability of an accident. Agents then face a trade-off between saving the cost of taking care and reducing the risk of being involved in an accident. We assume throughout that agents are rational and risk neutral, that is they minimize their *private* (expected) cost. The *social* cost is given by the aggregated private cost of care plus the *expected* cost of an accident, that is the probability of an accident multiplied by the damage the accident would cause. It shall be assumed that there is a unique level of care that minimizes the social cost: in this case the additional cost of any further damage prevention would be higher than the additional reduction of the expected cost of an accident. In order to minimize social cost government must incentivize agents to apply this care level; from here we will refer to it as the *standard of care*. On the other hand, the agents' objective is to choose their care level as to minimize their expected private cost. This poses a free-rider problem as each agent may rely on the other agents taking care and avoiding accidents. In particular, it is not clear at all that the social and private interests are aligned (see for instance Shavell, 1980; Landes and Posner, 1980; Kornhauser and Revesz, 2000). One way for the government to reconcile these interests is to combine Axioms 1 and 2 with an appropriate liability rule.

Axiom 1*. An agent has to pay a positive compensation payment if and only if (i) he caused a positive damage and (ii) his care level was lower than the standard of care.

Axiom 2*. The sum of the compensation payments covers exactly all damages that have been directly or indirectly caused by at least one agent with a care level below the standard of care.

Different from Axioms 1 and 2, the Axioms 1* and 2* now refer to a fixed standard of care. In this sense, they build a specification of the former, which allowed for other interpretations of negligence. The implied liability rule, together with the full recovery principle has already a very strong implication. To formulate it, we need the concept of *Nash equilibrium* (Nash, 1950): a Nash equilibrium is a list of choices, one choice for each player, such that no agent has an incentive to change their choice, given the choices of the others. Hence, a Nash equilibrium is a situation in which the agent's choices are self-sufficient; whenever the society is not in a Nash equilibrium, there is at least one agent who could unilaterally improve by choosing differently (provided the others stick to their choices).

Theorem 3.1. *In a society where the implemented allocation scheme satisfies Axioms 1* and 2*, there is only one Nash Equilibrium. In this Nash Equilibrium, each player will choose their care level equal to the standard of care.*

A direct consequence of this theorem is that the replacement of Axioms 1 and 2 in Theorems 2.1 and 2.2 by Axioms 1* and 2* leads to the characterizations of (unique) allocation schemes that create the desired incentives.

4 Settlements

For this section we take the allocation scheme that has been described in Section 2 as given. When one of the defendants settles, the liability situation is adjusted: the settling agent is removed and the remaining agents have to compensate for the original damage either net the settlement amount (pro tanto rule) or net the amount the allocation scheme would attribute to the settling agent (apportioned set-off rule). In the case of simultaneous tortfeasors this reduction is easily done, as the remaining damage is equally split between all agents who did not settle. If, however, there have been several subsequent torts, it is not clear at all what part of the settlement amount should be used in order to reduce how much of each incremental damage. A *settlement amendment scheme* is a list of weights, one for each damage, that add up to 100%. It serves as the key according to which any settlement amount is distributed between all the damages of the causality chain. So, any settlement leads to a new liability situation where the settling agent is removed from his groups, and the damage of each group is adjusted by that percentage of the settlement amount that is specified by the settlement amendment scheme.

Under the apportioned set-off rule one would expect the payments of non-settling agents in the new liability situation to be equal to the payments in the original liability situation. Under the pro tanto rule, one would expect the new payments of all non-settling agents to be higher whenever the settlement is lower than what is originally specified by the allocation rule. If a settlement amendment scheme achieves the latter, it *promotes settlements*. The emphasis here lies on *all* non-settling agents: if the settlement amount is lower than the allocation payment, it is clear that under the pro tanto there is at least *one* agent who will have to pay more. Requiring that this holds for *all* agents, however, pins down a unique settlement amendment scheme.

Theorem 4.1. *There is a unique settlement amendment scheme that promotes settlements. According to this scheme each incremental damage is adjusted as follows:*

1. *if the settling agent was neither directly nor indirectly responsible for it, it is not adjusted,*
2. *otherwise it is divided by the number of negligent agents who are directly or indirectly responsible for it (including the settling agent),*
3. *this per capita damage is multiplied by the ratio between the settling agent's settlement amount and the amount he would have to pay according to the allocation scheme,*
4. *the damage is reduced by this amount.*

We shall apply this settlement amendment scheme to our initial example in the case that one of the drivers settles at an amount of \$175.000. (A general formula for the weights in this settlement amendment scheme is provided in the appendix.) The driver is (directly or indirectly) responsible for all incremental damages. So, the per capita damages according to step 2 are \$50,000 and \$300,000. The driver settles at \$175.000 which is 1/2 of the amount specified by the allocation scheme. So, the first incremental damage is reduced by $1/2 \times \$50,000 = \$25,000$ and the second one by $1/2 \times \$300,000 = \$150,000$. The remaining agents face a new liability situation with incremental damages \$75,000 and \$750,000, which would lead to payments of \$450,000 and \$375,000.

Interestingly, this is also the only settlement amendment scheme that ensures that under the apportioned set-off rule the payments of non-settling agents in the original and the new liability situation are equal.

5 Conclusion

We have shown that the rigid use of reasonable and commonly accepted rules in the law of tort uniquely determines a scheme according to which compensation payments should be allocated to several tortfeasors. Even under the joint and several liability rule, these payments together with a unique settlement amendment scheme promote settlements. In this case there is a unique settlement amendment scheme that promotes settlements. The allocation schemes we provided further incentivize agents to apply the efficient standard of care. That means that they can be used to achieve a socially efficient outcome. We showed, however, that our allocation schemes are not the only ones that do so. Every allocation scheme that satisfies consistency with a rule of liability (which is defined by a standard of care) and ensures the recovery of all damages that have negligently been caused, will lead to an efficient outcome in the unique Nash equilibrium.

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Mathematical Appendix

A Proofs of Theorems 2.1 and 2.2

A *liability situation* is a triple $(N, \mathbf{S}, \Delta)$ where N is the (finite) set of tortfeasors, $\mathbf{S} = (S_k)_{k=1}^m$ is a vector of coalitions $S_k \subseteq N$ with $S_k \cap S_l = \emptyset$ for all $k \neq l$, and $\bigcup_{k=1}^m S_k = N$, and $\Delta = (\Delta_k)_{k=1}^m \in \mathbb{R}^m$ is a vector of non-negative damages. An *allocation scheme* is a map f that maps any liability situation $(N, \mathbf{S}, \Delta)$ on a vector $(f_i(N, \mathbf{S}, \Delta))_{i \in N} \in \mathbb{R}_{\geq 0}^N$ of *compensation payments*. For a liability situation $(N, \mathbf{S}, \Delta)$ denote by S_k^* the members of S_k who were negligent, let $N^* = \bigcup_{k=1}^m S_k^*$, and let

$$\Delta_k^* = \begin{cases} \Delta_k & \text{if } \bigcup_{l \leq k} S_l^* \neq \emptyset, \\ 0 & \text{otherwise} \end{cases}$$

be the damages that are caused (directly or indirectly) by at least one negligent agent. The mathematical formulation of the axioms 1-5 is as follows.

Axiom 1. For any liability situation $(N, \mathbf{S}, \Delta)$ and any $i \in N$ it holds that $f_i(N, \mathbf{S}, \Delta) > 0$ if and only if there is k such that $i \in S_k^*$ and $\sum_{l \geq k} \Delta_l > 0$.

Axiom 2. For any liability situation $(N, \mathbf{S}, \Delta)$ it holds that $\sum_{i \in N} f_i(N, \mathbf{S}, \Delta) = \sum_{k=1}^m \Delta_k^*$.

Axiom 3. For any liability situation $(N, \mathbf{S}, \Delta)$ and any two agents $i, j \in N$ with $i, j \in S_k^*$ for some $k = 1, \ldots, m$ it holds that $f_i(N, \mathbf{S}, \Delta) = f_j(N, \mathbf{S}, \Delta)$.

Axiom 4. If $(N, \mathbf{S}, \Delta)$ and $(N, \mathbf{S}, \Delta')$ are such that there is $k \leq m$ with $\Delta'_h = \Delta_h$ for all $l \geq k$, then $f_i(N, \mathbf{S}, \Delta) = f_i(N, \mathbf{S}, \Delta')$ for all $i \in \bigcup_{l=k}^m S_l$.

Axiom 5. For all liability situations $(N, \mathbf{S}, \Delta)$ with $\Delta_k = 0$ for some k it holds that

$$f(N, \mathbf{S}, \Delta) = f(N, \mathbf{S}_{-k}, \Delta_{-k}),$$

where

$$\begin{aligned} S_{-k} &= (S_1, \ldots, S_{k-1}, S_k \cup S_{k+1}, \ldots, S_m) \\ \Delta_{-k} &= (\Delta_1, \ldots, \Delta_{k-1}, \Delta_{k+1}, \ldots, \Delta_m). \end{aligned}$$

Any liability situation $(N, \mathbf{S}, \Delta)$ can be naturally associated with a characteristic function form game $\left(N, v^{N,\mathbf{S},\Delta}\right)$ by setting

$$v^{N,\mathbf{S},\Delta}(T) = \sum_{k: \bigcup_{l=1}^{k} S_l^* \subseteq T} \Delta_k^*.$$

for all $T \subseteq N$ (Dehez and Ferey, 2013). In particular, players who are not negligent are null players in this game.

Proof of Theorem 2.1. The proposed allocation scheme f is given by

$$f_i(N, \mathbf{S}, \Delta) = \begin{cases} \frac{\Delta_1^*}{|N^*|} & \text{if } i \in N^*, \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, f satisfies Axioms 1-3. Suppose there is another allocation scheme g that satisfies all three axioms as well. By Axiom 1 $g_i(N, \mathbf{S}, \Delta) = 0 = f_i(N, \mathbf{S}, \Delta)$ for all $i \notin N^*$. If there is at least one $i \in N^*$ then, by Axiom 2, $\sum_{i \in N^*} g_i(N, \mathbf{S}, \Delta) = \Delta_1^* = \Delta_1$. Hence, by Axiom 3, $g_i(N, \mathbf{S}, \Delta) = \frac{\Delta_1}{|N^*|} = f_i(N, \mathbf{S}, \Delta)$. Hence, f and g coincide. *Q.E.D.*

Proof of Theorem 2.2. We have to prove that the allocation scheme f defined by

$$f_i(N, \mathbf{S}, \Delta) = \sum_{k: i \in \bigcup_{l=1}^{k} S_k^*} \frac{\Delta_k^*}{\left|\bigcup_{l=1}^{k} S_k^*\right|}.$$

for all $i \in N$ is the only allocation scheme that satisfies axioms 1 to 5. It can easily be seen that

$$f(N, \mathbf{S}, \Delta) = Sh\left(N, v^{N,\mathbf{S},\Delta}(S)\right), \tag{1}$$

where Sh is the *Shapley value* Shapley (1953b). Hence, Axiom 1-3 follow from the null player property, efficiency, and symmetry of the Shapley value. Axiom 5 holds as the two liability situations $(N, \mathbf{S}, \Delta)$ and $(N, \mathbf{S}_{-k}, \Delta_{-k})$ are associated with the same characteristic function form game. Axiom 4 is satisfied because of the strong monotonicity of the Shapley value (Young, 1985) as $v^{N,\mathbf{S},\Delta}(S) - v^{N,\mathbf{S},\Delta}(S \setminus \{i\}) = v^{N,\mathbf{S},\Delta'}(S) - v^{N,\mathbf{S},\Delta'}(S \setminus \{i\})$ for all such i.

For the uniqueness of f suppose that there is another allocation scheme g that satisfies the axioms as well. For any liability situation $(N, \mathbf{S}, \Delta)$ let $I(\Delta) = |\{k : \Delta_k > 0\}|$. If $I(\Delta) = 0$, then there are no positive damages, and Axiom 1 implies that $g_i(N, \mathbf{S}, \Delta) = 0 = f_i(N, \mathbf{S}, \Delta)$ for all $i \in N$. Let $(N, \mathbf{S}, \Delta)$ be such that $I(\Delta) \geq 1$, and that the claim is true for all liability situations $(N, \mathbf{S}', \Delta')$ with $I(\Delta') < I(\Delta)$. By Axiom 5 we can assume without loss of generality that $\Delta_1 > 0$. Define $(N, \mathbf{S}, \Delta')$ by $\Delta'_1 = 0$ and $\Delta'_k = \Delta_k$ for all $k \geq 2$. Then

$$g_i(N, \mathbf{S}, \Delta) = g_i(N, \mathbf{S}, \Delta') = f_i(N, \mathbf{S}, \Delta') = f_i(N, \mathbf{S}, \Delta)$$

for all $i \in \bigcup_{l=2}^{m} S_l$ by Axiom 4 and the induction hypothesis. Hence, $g_i(N, \mathbf{S}, \Delta) =$

$f_i(N, \mathbf{S}, \Delta)$ for all $i \in S_1$ by Axioms 2, and 3. *Q.E.D.*

For the independence of the axioms note that the allocation scheme $f_i^1(N, \mathbf{S}, \Delta) = \sum_{k: i \in \bigcup_{l=1}^k S_l} \frac{\Delta^*}{\left|\bigcup_{l=1}^k S_l\right|}$ satisfies all axioms but Axiom 1. Further, $f^2(N, \mathbf{S}, \Delta) = \frac{1}{2} f(N, \mathbf{S}, \Delta)$ satisfies all axioms but Axiom 2. If the Shapley value in Equation (1) is replaced by a *weighted* Shapley value (Shapley, 1953a; Kalai and Samet, 1987) one obtains an allocation scheme that satisfies all axioms but Axiom 3. The allocation scheme

$$f_i^4(N, \mathbf{S}, \Delta) = \begin{cases} \frac{1}{\left|\bigcup_{k=1}^m S_k^*\right|} \sum_{k=1}^m \Delta_k^*, & \text{if } i \in S_k^* \text{ for some } k = 1, \ldots, m \\ 0, & \text{otherwise} \end{cases}$$

satisfies all axioms but Axiom 4. Let m^* be the number of sets S_k that contain at least one negligent agent. The allocation scheme

$$f_i^5(N, \mathbf{S}, \Delta) = \sum_{k: i \in \bigcup_{l=1}^k S_l^*} \frac{1}{\left|\bigcup_{l=1}^k S_l^*\right|} \left(\frac{1}{m^*} \sum_{k=1}^m \Delta_k^* \right)$$

satisfies all axioms but Axiom 5.

B Proof of Theorem 3.1

Let $(N, \mathbf{S}, \Delta)$ be a (fixed) liability situation. Let $x_i \in \mathbb{R}$ be agent i's level of care and denote by $C_i(x_i)$ the associated private cost of agent i, where C_i is an increasing function. By $p_k\left((x_i)_{i \in S_k}\right)$ denote the probability that group S_k causes damage Δ_k provided that each $i \in S_k$ chooses x_i as his level of care; let p_k be decreasing in all coordinates. Then the expected social costs of the liability situation are given by

$$SC(x) = \sum_{i \in N} C_i(x_i) + \sum_{k=1}^m p_k\left((x_i)_{i \in S_k}\right) \Delta_k. \tag{2}$$

Assume that SC has a unique minimum,[1] and denote the minimizer of SC by $x^* \in \mathbb{R}^N$. We interpret x_i^* as the standard of care that applies to agent i; in particular, different standards of care may apply to different agents in the liability situation. The adapted axioms then read as follows.

Axiom 1*. For any liability situation $(N, \mathbf{S}, \Delta)$ and any $i \in N$ it holds that $f_i(N, \mathbf{S}, \Delta) > 0$ if and only if $x_i < x_i^*$ and $\sum_{k: i \in \bigcup_{l=1}^k S_k} \Delta_k > 0$.

Axiom 2*. For any liability situation $(N, \mathbf{S}, \Delta)$ it holds that $\sum_{i \in N} f_i(N, \mathbf{S}, \Delta) = \sum_{k=1}^m \Delta_k^*$, where

$$\Delta_k^* = \begin{cases} \Delta_k & \text{if there is } i \in \bigcup_{l \leq k} S_l \text{ with } x_i < x_i^*, \\ 0 & \text{otherwise.} \end{cases}$$

Proof of Theorem 3.1. We first show that x^* is a Nash equilibrium. It is obvious that no agent has an incentive to choose $x_i > x_i^*$. Suppose that all agents $j \in N \setminus \{i\}$ chose $x_j = x_j^*$ and assume that $x_i < x_i^*$. Then agent i's expected payment is

$$C_i(x_i) + \sum_{k: i \in \bigcup_{l=1}^k S_l} p_k\left((x_j)_{j \in S_k}\right) \Delta_k.$$

as i has to recover the full damage alone. As x^* is the unique minimizer of the social cost function in Equation (2) it holds that

$$\begin{aligned} SC(x^*) &= \sum_{j \in N} C_j\left(x_j^*\right) + \sum_{k=1}^m p_k\left(\left(x_j^*\right)_{j \in S_k}\right) \Delta_k \\ &< \sum_{j \neq i} C_j\left(x_j^*\right) + C_i(x_i) + \sum_{k: i \notin \bigcup_{l=1}^k S_l} p_k\left(\left(x_j^*\right)_{j \in S_k}\right) \Delta_k \\ &\quad + \sum_{k: i \in \bigcup_{l=1}^k S_l} p_k\left(x_i, \left(x_j^*\right)_{j \in S_k \setminus \{i\}}\right) \Delta_k \end{aligned}$$

[1] Existence and uniqueness can be guaranteed for instance if C_i is convex for all i and p_k is strictly convex.

and therefore

$$\begin{aligned} C_i\left(x_i\right)+\sum_{k:i\in\bigcup_{l=1}^{k}S_l} p_k\left(x_i,\left(x_j^*\right)_{j\in S_k\setminus\{i\}}\right)\Delta_k &> C_i\left(x_i^*\right)+\sum_{k:i\in\bigcup_{l=1}^{k}S_l} p_k\left(\left(x_j^*\right)_{j\in S_k}\right)\Delta_k \\ &\geq C_i\left(x_i^*\right). \end{aligned}$$

Hence, x_i imposes higher expected costs on i than x_i^*, so x^* is a Nash Equilibrium.

We now show that x^* is, in fact, the only Nash Equilibrium. For this purpose let x be vector of care levels, and assume that x is a Nash Equilibrium. Let A be the set of agents who choose $x_j = x_j^*$, and let $B = N \setminus A$ the set of agents who choose $x_i < x_i^*$. (Recall that no agent would choose $x_j > x_j^*$ in a Nash Equilibrium.) Then, by Axioms 1^* and 2^*, the total expected costs that the agents in B have to bear are

$$\sum_{i\in B} C_i\left(x_i\right)+\sum_{k:\bigcup_{l=1}^{k}S_l\cap B\neq\emptyset} p_k\left(\left(x_j^*\right)_{j\in A\cap S_k},\left(x_i\right)_{i\in B\cap S_k}\right)\Delta_k.$$

(Recall that $\Delta_k = \Delta_k^*$ for all k with $\bigcup_{l=1}^{k} S_l \cap B \neq \emptyset$.) By the definition of x^* we have $SC\left(x^*\right) < SC\left(x\right)$ and therefore,

$$\begin{aligned} &\sum_{i\in B} C_i\left(x_i\right)+\sum_{k:\bigcup_{l=1}^{k}S_l\cap B\neq\emptyset} p_k\left(\left(x_j^*\right)_{j\in A\cap S_k},\left(x_i\right)_{i\in B\cap S_k}\right)\Delta_k \\ &\quad > \sum_{i\in B} C_i\left(x_i^*\right)+\sum_{k:\bigcup_{l=1}^{k}S_l\cap B\neq\emptyset} p_k\left(\left(x_i^*\right)_{i\in S_k}\right)\Delta_k \\ &\quad \geq \sum_{i\in B} C_i\left(x_i^*\right). \end{aligned}$$

Since the aggregated expected costs of the agents in B are strictly greater if they choose $\left(x_i^*\right)_{i\in B}$ than if they choose $\left(x_j^*\right)_{j\in B}$, there must be at least one agent $i \in B$ whose private expected costs are strictly greater from choosing x_i than from choosing x_i^*. Hence, x cannot be a Nash Equilibrium. *Q.E.D.*

C Proof of Theorem 4

A settlement amendment scheme is a vector $(r_k)_{k=1}^m$ with $r_k \geq 0$ for all $k = 1, \ldots, m$ and $\sum_{k=1}^m r_k = 1$. If agent i settles and pays an amount θ, the remaining agents face the new liability situation $(N \setminus \{i\}, \mathbf{S}', \Delta')$ with

$$\begin{aligned} S'_k &= S_k \setminus \{i\} \\ \Delta'_k &= \Delta_k - r_k \theta. \end{aligned}$$

The settlement amendment scheme r promotes settlements if

$$f_j (N \setminus \{i\}, \mathbf{S}', \Delta') > f_j (N, \mathbf{S}, \Delta)$$

for all $j \neq i$ if and only if $\theta < f_i (N, \mathbf{S}, \Delta)$.

Proof of Theorem 4.1. The proposed settlement amendment scheme is given by

$$r_k = \begin{cases} \frac{\Delta_k^*}{\left|\bigcup_{l=1}^k S_k^*\right|} \frac{1}{f_i(N,\mathbf{S},\Delta)} & \text{if } i \in \bigcup_{l=1}^k S'_l \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to verify that r has the desired property; in fact, this follows from the consistency of the Shapley value (Hart and Mas-Colell, 1989).

Let r promote settlements and let i be an agent with $f_i (N, \mathbf{S}, \Delta) > 0$ who settles at θ. Then $f_j (N \setminus \{i\}, \mathbf{S}', \Delta') \leq f_j (N, \mathbf{S}, \Delta)$ for all $j \neq i$ if and only if $\theta \geq f_i (N, \mathbf{S}, \Delta)$. In case that $\theta = f_i (N, \mathbf{S}, \Delta)$ one further obtains

$$\sum_{j \neq i} f_j (N, \mathbf{S}', \Delta') = \sum_{k=1}^m \Delta_k^* - \theta = \sum_{k=1}^m \Delta_k^* - f_i (N, \mathbf{S}, \Delta) = \sum_{j \neq i} f_j (N, \mathbf{S}, \Delta).$$

Hence, in this case it must hold that $f_j(N \setminus \{i\}, \mathbf{S}', \Delta') = f_j(N, \mathbf{S}, \Delta)$ for all $j \neq i$. Using the definitions of f and $(N \setminus \{i\}, \mathbf{S}', \Delta')$ this leads to

$$\sum_{k:j\in\bigcup_{l=1}^{k} S_l} \frac{\Delta_k^*}{\left|\bigcup_{l=1}^{k} S_l\right|} = f_j(N, \mathbf{S}, \Delta) = f_j(N \setminus \{j\}, \mathbf{S}', \Delta') = \sum_{k:j\in\bigcup_{l=1}^{k} S_l} \frac{\Delta_k^* - r_k f_i(N, \mathbf{S}, \Delta)}{\left|\bigcup_{l=1}^{k} S_l \setminus \{i\}\right|}$$

or equivalently

$$\sum_{k:j\in\bigcup_{l=1}^{k} S_l} \frac{r_k f_i(N, \mathbf{S}, \Delta)}{\left|\bigcup_{l=1}^{k} S_l \setminus \{i\}\right|} = \sum_{k:j\in\bigcup_{l=1}^{k} S_l} \left(\frac{\Delta_k^*}{\left|\bigcup_{l=1}^{k} S_l \setminus \{i\}\right|} - \frac{\Delta_k^*}{\left|\bigcup_{l=1}^{k} S_l\right|} \right) \tag{3}$$

for all $j \neq i$. If $S_1 \neq \{i\}$ these are m linear equations that are linearly independent (as the system is triangular and all diagonal entries are strictly positive), so the solution is unique. If $S_1 = \{i\}$ the requirement that $\sum_{k=1}^{m} r_k = 1$ is another constraint that is linearly independent of the $m-1$ independent (non-trivial) equations in (3). Hence, in both cases the solution is unique. As the proposed settlement amendment scheme r solves this linear equation system, it is the only solution. *Q.E.D.*